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Falling birth rates and world population decline: A quantitative discussion (1950-2040)

Félix-Fernando MUÑOZ[1*] & Julio A. GONZALO[2]

Abstract: The UN data (1950-2010) and projections (both medium and low-fertility variants for 2015-2040) show that fertility rates are already below replacement level in all continents except Africa. In this paper we develop a simple new approach for population projections based on a Improved Rate Equations (IRE) model. Population projections under the (1) Malthusian assumption, (2) an (IRE) model fitting and extrapolating from actual UN population data up to 2040, and (3) UN projections (low-fertility variant), are compared. The model fits quite well actual data and suggests a world population decline in the 21st Century. The economic, social and political consequences of this new and global circumstance would be far reaching.

Keywords: *world population decline, population trends, jump in population level, improved rate equations model*

JEL: J10, J11.

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1. Introduction

Although demographics is a determining factor in explaining the long-term development of an economy -together with ‘the stock of human knowledge particularly as applied to the human command over nature; and the institutional framework that defines the deliberate incentive structure of a society’- (North, 2005, p. 1; see also Sachs, 2002), it is usually understated in economic growth theory. Population long-term dynamics is often described in a very simplified or mechanistic way: in fact, population dynamics tends to be represented by means of simple dynamic models. In this sense, it is quite usual to consider the evolution of population in economic models under a Malthusian growth dynamics assumption.[1]

In this paper we present a quantitative discussion of world population prospects which questions Malthusian dynamics. Extrapolating UN data (available since 1950), it would be expected that world population stagnates because of the decline in birth rates presently observable both globally and by continents. According to the population database of the Population Division of the Department of Economic and Social Affairs of the UN Secretariat,[2] fertility rates are below replacement levels in all continents except Africa. Because of the momentum gained in previous decades, world population and continental population in Africa, the Americas (Northern America and Latin America), Asia and Europe are still increasing at a slow rate, but demographers see signs of decrease in world population somewhere around the middle of the 21st Century (Chaunu, 1997; Eberstadt, 2001).

[1] In (endogenous) economic growth literature (see for example Acemoglu (2009)), economic growth models usually assume that labor (as an approximation for population) increases following the rule $L_t = L_0 e^{nt}$, with $L_0, n > 0$. This is a typical Malthusian (Malthus, 1985 [1798]) dynamics.

[2] Data source: UN Population Division (http://www.un.org/esa/population/unpop.htm). For the years 1920, 1930, 1940 and 1950 the source is UN (1959) *Demographic Yearbook*, New York, p. 127.

The implications of the expected evolution of population are far reaching from an economic point of view –and indeed from *any* point of view: social, political, geo-strategic, environmental, etc. (Ulrich, 2000; Yea, 2004). And the reason is not only population stagnation but, to a great extent, the change in the age composition of human population. For the first time in history humanity as a whole confronts ageing as a global problem.

In order to simulate population trends, we present in this paper a very simple mathematical model using a hyperbolic tangent solution fitting effectively any *jump in population level*. The Improved Rate Equation (IRE) solutions are used, as far as we know, for the very first time to model population dynamics and it is one of the main contributions of this article.[3] The other main contribution consists of applying this (IRE) to the world as a whole. Previously, a rate equations approach has been recently proposed by the authors to describe steps up or down in population due to changes in birth rate (*BR*), death rate (*DR*) or both (see Gonzalo, Muñoz, & Santos, 2013). A step up in population between a given *starting* population level $\left(P_{RL}\right)_n$ (replacement level) and a certain *final* population level $\left(P_{RL}\right)_{n+1}$, -where *P* means 'population', *RL* 'replacement level' and *n* and *n* + 1 refer, respectively, to the initial level and to the final level for which population dynamics evolves at a certain rate.[4] In that previous model, population dynamics was determined by the (*BR*) extracted from the fertility rate (*FR*), and the (*DR*) extracted from the combination of the death rate (*DR*) and the growth rate (*GR*) (Ibid., pp. 194-195). Suppose that at a given replacement level *RL*, $FR \simeq 2.1$ and $BR \simeq DR$. Since the UN data show that the overall (*FR*) (world and individual continents) has decreased consistently in the period 1950-2010, we assume that the long-term initial $\left(DR\right)_n$ -compensated for a $\left(BR\right)_n \simeq \left(DR\right)_n$ - is higher than the final $\left(DR\right)_{n+1}$ -compensated for a $\left(BR\right)_{n+1} < \left(BR\right)_n$ in the long run. Under these assumptions, population dynamics would be essentially governed by an increase in life expectancy. However, increases in life expectancy cannot go on forever.

[3] An excellent survey on population dynamics is provided by Bacaër (2011). See also Miranda and Lima (2010; 2011). The (IRE) model (see Eq. 1 bellow) is a certain improvement over the simple rate equations solution given in (Gonzalo et al., 2013)

[4] In other words, (*RL*) means that birth rates and death rates are equal at that instant of time.

The structure of the paper is as follows. Section 2 is devoted to data analysis globally and by continents. Section 3 shows and compares the projections of world population of UN and those obtained with the IRE model. Section 4 discusses the main findings and implications of the model.

2. Data analysis

UN data (1950-2010) show a strong proportionality between the birth rate the birth rate (*BR*) and fertility rate (*FR*). Table 1 shows how consistent is the proportionality between (*BR*) and (*FR*) for the world and the various continents.[5]

[Table 1]

Table 2 gives the respective fertility rate (*FR*), growth rate (*GR*), birth rate (*BR*), death rate (*DR*) for Africa, Asia, Europe, Northern America, Latin America, Oceania and the World respectively. From these data it is possible to calculate the value of the variables $x = BR/DR$, $y = BR \cdot DR$, $\alpha = \frac{1}{2}\mathrm{Ln}x$, $\tau^{-1} = 2y^{1/2} \cdot 10^{-2}$ and τ (characteristic time) for the years 1950-55 to 2005-2010, adding the extrapolated UN data –using UN medium-fertility variant hypothesis- for 2010-15.[6] In all cases, the raw UN data are (*FR*), (*BR*), (*DR*) and (*GR*). Variables x, y, α, τ^{-1}, and τ are calculated as defined above.

[Table 2]

Figure 1 is a representation of (*BR*) vs. (*FR*), for the Europe and Northern America (1a), Africa and Latin America (1b), and Asia and the World (1c). The patterns are very similar and a quick reduction of the relationship between (*BR*) and (*FR*) is observed in all continents. Figure 2a and 2b show the behavior of $x(t)$ and $\tau(t)$ respectively.[7] Time evolution of $x(t)$ and $\tau(t)$ by continents speak for themselves.

[Figure 1]

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[5] The proportionality factors for each continent have been determined by means of a representative sample of the most populous countries in each continent.
[6] Along the paper we use usually UN projections under low-fertility variant because historically this variant has been more consistent with actual data as time went by.
[7] For simplicity, in the rest of the paper we omit references to Oceania.

[Figure 2a]

[Figure 2b]

Table 3 gives the world population $P_{UN}(t)$ -UN data- for the period 1950-2040, along with the calculated population $P_{cal}(t)$ using the (IRE) model, where:

$$P_{cal}(t) = P_{RL} + \frac{1}{2}\Delta P_m(\tanh\alpha)\left(1 + \tanh\frac{t - t_{\inf}}{\tau^*}\right) \qquad (1)$$

and $\Delta P_m(\tanh\alpha)$, $t_{\inf}$ and τ^* represent, respectively the jump amplitude -that depends on the values of $(RL)_n$ and $(RL)_{n+1}$ (in this case 5.86 billions)-, the inflection point (at year 1985), and the characteristic time -that is somehow related to the average fertility age for women.

[Table 3]

In order to fit the function to current data and compute the associated projection, we use $P_{RL}(1960) \simeq 2.0 \times 10^9$, $t_{\inf} = 1985$ and $\tau^* = 32$ years. Additionally, Eq. (1) makes use of the next approximation:

$$\frac{1}{2}(1 + \tanh x) \simeq \frac{1}{2}\left(1 + \left(1 - 2e^{-2x} + e^{-4x}\right)\right) = e^{-2\tau x}\left(1 - e^{-2(x - \delta x)}\right) \qquad (2)$$

where $e^{-2\tau x} \equiv \left(1 + \frac{1}{2}e^{-4x}\right)$ for x in the interval $0 \le x \le \infty$. This is equivalent to the previous (more abrupt) rate equation solution (Gonzalo et al., 2013) where $t_{\inf} = 1965$ (instead of $t_{\inf} = 1985$) and $\tau^* = 27.2$ years (instead of $\tau^* = 32$ years).

Figure 3 shows the good fit obtained for the time span 1950-2040.

[Figure 3]

Table 4 gives in parallel columns the death rates (*DR*) and the estimated inverse female life expectancy of females $\left(LE_f\right)^{-1}$ for Africa, Europe, Asia, the Americas, and the World (1950-2010) -extrapolated from actual data. There seems to be a minimum inverse life expectancy around $\left(LE_f\right)^{-1} = 1.2\times10^{-2}$ which corresponds to a maximum $\left(LE_f\right) = 83.3$ years. The world data follow a straight line going through the origin. In fact, a zero death rate (*DR*) should correspond to an 'infinite life expectancy'. The data for Europe seem to undergo a time reversal at about $\left(LE_f\right)^{-1} = 1.20\times10^{-2}$.[8] Data in Table 4, corresponding to 1950-2010, are depicted in Figure 4.[9]

[Table 4]

[Figure 4]

A comparison between the time evolution of (*BR*) and (*DR*) for Europe and the World is given in Table 5 from 1950 to 2040.[10] The behavior of (*BR*) (related to the fertility rate) determine the trend of the population evolution in both cases. The *momentum* in net growth rate (*GR*) due to normal fertility in previous generations does not show yet an actual decrease with time in the resulting net population for the world (*WBR* is still larger than *WDR*), but for Europe, after a long period of increasing 'life expectation', the death rate is already going up, and since about 2000, it has surpassed the declining birth rate (*EDR* > *EBR*). Sometime around the middle of this century the same thing is likely to happen for the world as a whole. This is illustrated in Figure 5.

[Table 5]

[Figure 5]

3. Trends and projections

Since the early sixties of last century an exponential growth of world population was assumed in many influential circles including the UN and most Western governmental circles.[11] Even most recently, after signs to the contrary are increasingly evident,

[8] According to UN data (1990-2010), Japan (a large rich and ageing country) shows a similar time reversal.
[9] More direct data for death rate and life expectancy might show perhaps a little less statistical scatter; however the general trend is clear.
[10] From 2010-2040 we represent UN projections for low-fertility variant.
[11] The Club of Rome was the most famous and perhaps most influential one. See also the interest of *Science* on this topic: (Aldrich, 1968; Caldwell, 2008; Dorn, 1962; Holden, 1974, 1984; Horiuchi, 1992; Mead, 1974; Sachs, 2002; Sax, 1969).

respected media personalities -like Stephen Hawking- insist on the spectre of exponential growth. It is straightforward to construct a Malthusian projection for 1985-2040 using $GR = 1.85$ that corresponds to the increase of world population between 1985-1990 by means of

$$P_{Malthus}\left(t\right)_{n+1} = P\left(t\right)_n \left(1+\left(GR\right)_n\right)^5 \qquad (3)$$

For 2010, the Malthusian projection gives a world population of 7.60×10^9 -instead of the actual population of 6.90×10^9 - and would continue growing exponentially.[12]

Table 6 gives the numerical data for these projections. Figure 6 depicts these results for the period 1950-2010 (actual data) together with those of the Malthusian projection, the (IRE) projection and UN projection (low fertility variant, the more realistic) for (2010-2040).[13]

[Figure 6]

4. Discussion

The sustained reproductive capability of the human race is clearly much larger than that of other higher animals on Earth's surface. Today, *homo sapiens* number about 7×10^9 individuals scattered all over the planet while only about 3×10^4 individuals of the species orangutan, gorilla and chimpanzee populate some parts of Asia and Africa, after fifteen, ten and five million years, respectively, of their appearance on Earth. Our planet has considerable potential reserves of food, water and energy to house a human population estimated as some twenty or thirty times larger than the present one. Nevertheless, these potential reserves are finite.

The main findings of our approach point out to the following:

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[12] Assuming a growth rate of 0.0185 and departing from actual population in 1985.

[13] It would be very interesting to analyze whether with pre-1965 pre-chemical contraceptives, pre-legalized abortion, and modest 'spontaneous' decrease in fertility rates, world population would have leveled out at mid-21st Century. However, this exercise is out of the scope of this paper.

1. A continuous exponential world population growth is totally unrealistic. Rather, steps up (alternated with relatively stable periods or step down) should be expected. Usually, an increase in life expectancy (*LE*) is followed by a gradual decrease in fertility rate (*FR*). However, certain drastic policies promoting contraception and abortion (as is the case of China) may result in unwanted population trends with deleterious human, social and economic consequences.[14]

2. It is interesting to note that the local population density in various countries with annual *per capita* income in the range 15,000 to 45,000 USD is between five times and fifteen times higher than the present average world population.[15]

3. Taking into account that: (a) the replacement level fertility rate is about $FR = 2.1$ child per woman; (b) the birth rate is of the order of $BR = (0.75)FR$; (c) the life expectancy at fertility age (about 18 years), $(LE_{fa}) = (LE_b) - 18$, and for cases like Europe and Japan, for which population has leveled out already (and is even decreasing) as shown above, $(LE_b) \simeq (DR)/(0.833))$, population replacement level $(BR = DR)$ is attained when

$$BR = [(0.75) \times 2.1] \times 10^{-2} = 1.57 \times 10^{-2} = DR \qquad (4)$$

corresponding to

$$(LE_b) = 18 + (DR)/(0.833) = 70.8 \text{ years} \qquad (5)$$

An increase in women's life expectancy beyond 70.8 years is unlikely to result in an increase of birth rate.

4. Finally, graphs of population data of steps up in population for all continents could be fitted by means of our (IRE) (Eq. 1) for Africa, Europe, Asia, Northern America and Latin America. To this end, first the inflection point (corresponding to the maximum

[14] An example of this concern is a recent report urging a change in China's one child policy. See *The Guardian*, 31st October 2012, 'China think-tank urges end of one-child policy. Foundation close to central leadership urges end to birth limits policy across China by 2015, with experts saying reform is 'inevitable'.' The cited think-tank is *China Development Research Foundation* (http://www.cdrf.org.cn/en/).

[15] For instance: in UK, 5 time larger; in the Netherlands, 8 times; in Puerto Rico, 9 times; in South Korea, 10 times; in Taiwan, 13 times. Not counting exceptional cases like Hong Kong (with 6.4 million inhabitants), 126 times and Singapore (with 7.0 million inhabitants) which has 139 times the world average.

slope) for the step up in population should be estimated. This would allow us to get $t_{\inf}$, $P(t_{\inf})$, P_{RL}, and ΔP_m^* for each continent. Then τ^* is determined by means of

$$\tau^* = (\mp \Delta t) / \tanh^{-1}\left(\frac{2P(\mp \Delta t) - P_{RL}}{\Delta P_m^*} \right) \tag{6}$$

Where the data for $-\Delta t$ is well known, in principle, for all continents while data for $+\Delta t$ is not for Africa, Europe, Asia, Northern America, Latin America and Oceania. If data for $-\Delta t$ and $+\Delta t$ were available it is possible to calculate an average $\tau^* = \frac{1}{2}\left[\tau^*(-\Delta t) + \tau^*(+\Delta t)\right]$. If not, it is still possible to use $\tau^* \simeq \tau^*(-\Delta t)$.

Finally, in this paper we have focused in the role of inertia for the demographic evolution. An investigation of world population trends and their implications from other perspectives deserves special attention.

TABLES

Table 1: The relationship between birth rates and fertility rates by continents.

Africa	*BR* = 0.7114 *FR*
Europe	*BR* = 0.7502 *FR*
Asia	*BR* = 0.7267 *FR*
Northern America	*BR* = 0.7205 *FR*
Latin America	*BR* = 0.7334 *FR*
Oceania	*BR* = 0.7216 *FR*
World	***BR* = 0.7394 *FR***

Table 2: Regional and World population data: 1950-2015.*

Table 2a: Africa

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	*α*	τ^{-1}	τ
1950-55	6.60	2.114	4.7735	2.71	2.26	12.94	0.41	0.07	13.90
1955-60	6.66	2.305	4.7656	3.21	1.48	15.30	0.20	0.08	12.78
1960-65	6.71	2.439	4.7361	3.26	1.45	15.44	0.19	0.08	12.72

1965-70	6.68	2.560	4.6578	3.14	1.48	14.63	0.20	0.08	13.07
1970-75	6.67	2.651	4.6123	2.88	1.60	13.28	0.24	0.07	13.72
1975-80	6.57	2.772	4.5478	2.57	1.77	11.69	0.29	0.07	14.63
1980-85	6.39	2.797	4.4328	2.39	1.85	10.59	0.31	0.07	15.36
1985-90	6.07	2.692	4.2527	2.18	1.95	9.27	0.33	0.06	16.42
1990-95	5.62	2.529	4.0209	1.81	2.22	7.28	0.40	0.05	18.53
1995-2000	5.23	2.357	3.837	1.91	2.01	7.33	0.35	0.05	18.47
2000-05	4.94	2.326	3.7133	1.76	2.11	6.54	0.37	0.05	19.56
2005-10	4.64	2.301	3.5587	1.15	3.09	4.09	0.56	0.04	24.72
2010-15	4.37	2.274	3.4039	1.37	2.48	4.66	0.46	0.04	23.15

Table 2b: Asia

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	α	τ^{-1}	τ
1950-55	5.82	1.978	4.18	2.57	1.63	10.74	0.24	0.07	15.25
1955-60	5.58	1.947	3.92	2.35	1.67	9.22	0.26	0.06	16.46
1960-65	5.59	1.988	3.85	2.12	1.82	8.16	0.30	0.06	17.50
1965-70	5.61	2.478	3.81	1.95	1.96	7.44	0.34	0.05	18.34
1970-75	5.00	2.282	3.46	1.64	2.11	5.67	0.37	0.05	21.00
1975-80	4.05	1.946	2.96	1.33	2.23	3.94	0.40	0.04	25.19
1980-85	3.69	1.944	2.88	0.96	3.00	2.76	0.55	0.03	30.08
1985-90	3.44	1.918	2.80	0.76	3.69	2.13	0.65	0.03	34.26
1990-95	2.97	1.626	2.48	0.69	3.60	1.71	0.64	0.03	38.22
1995-2000	2.65	1.384	2.20	0.68	3.25	1.50	0.59	0.02	40.88
2000-05	2.41	1.180	1.98	0.69	2.85	1.37	0.52	0.02	42.70
2005-10	2.28	1.082	1.86	0.68	2.72	1.27	0.50	0.02	44.35
2010-15	2.18	0.990	1.75	0.70	2.51	1.22	0.46	0.02	45.29

Table 2c: Europe

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	α	τ^{-1}	τ
1950-55	2.65	0.996	2.144	1.07	2.00	2.30	0.35	0.03	32.96
1955-60	2.64	0.971	2.072	1.07	1.93	2.22	0.33	0.03	33.53
1960-65	2.56	0.961	1.908	1.04	1.83	1.99	0.30	0.03	35.48
1965-70	2.35	0.691	1.672	1.15	1.45	1.92	0.19	0.03	36.06
1970-75	2.17	0.608	1.559	1.08	1.44	1.68	0.18	0.03	38.54
1975-80	1.98	0.489	1.476	1.03	1.43	1.52	0.18	0.02	40.56
1980-85	1.89	0.398	1.434	1.06	1.35	1.52	0.15	0.02	40.56
1985-90	1.82	0.384	1.370	0.97	1.41	1.33	0.17	0.02	43.37
1990-95	1.57	0.191	1.151	1.07	1.08	1.23	0.04	0.02	45.16
1995-2000	1.42	-0.018	1.024	1.08	0.95	1.11	-0.03	0.02	47.55
2000-05	1.43	0.109	1.015	1.14	0.89	1.16	-0.06	0.02	46.48
2005-10	1.53	0.203	1.076	1.18	0.91	1.27	-0.05	0.02	44.38
2010-15	1.59	0.105	1.084	1.24	0.87	1.34	-0.07	0.02	43.13

Table 2d: Northern America

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	α	τ^{-1}	τ
1950-55	3.33	1.713	2.46	1.13	2.17	2.78	0.39	0.03	30.01

1955-60	3.64	1.776	2.46	1.32	1.86	3.24	0.31	0.04	27.77
1960-65	3.36	1.408	2.20	1.13	1.95	2.49	0.33	0.03	31.69
1965-70	2.55	1.071	1.77	1.01	1.76	1.79	0.28	0.03	37.35
1970-75	2.05	0.936	1.57	1.14	1.38	1.79	0.16	0.03	37.36
1975-80	1.80	0.974	1.51	0.47	3.21	0.71	0.58	0.02	59.41
1980-85	1.79	0.969	1.54	0.47	3.28	0.73	0.59	0.02	58.70
1985-90	1.87	1.028	1.55	0.53	2.93	0.82	0.54	0.02	55.14
1990-95	1.96	1.012	1.51	0.86	1.76	1.30	0.28	0.02	43.85
1995-2000	1.93	1.152	1.39	0.85	1.64	1.18	0.25	0.02	46.00
2000-05	1.99	0.993	1.37	0.88	1.56	1.21	0.22	0.02	45.46
2005-10	2.03	0.908	1.37	0.84	1.63	1.15	0.24	0.02	46.61
2010-15	2.04	0.858	1.35	0.82	1.64	1.11	0.25	0.02	47.56

Table 2e: Latin America

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	α	τ^{-1}	τ
1950-55	5.86	2.715	4.27	2.38	1.79	10.15	0.29	0.06	15.69
1955-60	5.92	2.758	4.19	2.38	1.76	9.97	0.28	0.06	15.83
1960-65	5.96	2.759	4.11	2.36	1.74	9.69	0.28	0.06	16.06
1965-70	5.53	2.510	3.78	2.17	1.74	8.22	0.28	0.06	17.44
1970-75	5.02	2.411	3.51	1.85	1.90	6.49	0.32	0.05	19.62
1975-80	4.47	2.293	3.30	1.51	2.19	4.98	0.39	0.04	22.41
1980-85	3.93	2.098	3.07	1.23	2.50	3.78	0.46	0.04	25.73
1985-90	3.42	1.924	2.79	1.00	2.80	2.78	0.51	0.03	29.99
1990-95	3.02	1.713	2.53	0.85	2.97	2.16	0.54	0.03	34.04
1995-2000	2.73	1.546	2.32	0.76	3.03	1.77	0.55	0.03	37.58
2000-05	2.53	1.321	2.14	0.73	2.94	1.56	0.54	0.02	40.04
2005-10	2.30	1.153	1.93	0.72	2.69	1.38	0.49	0.02	42.49
2010-15	2.17	1.068	1.78	0.75	2.39	1.33	0.44	0.02	43.40

Table 2f: Oceania

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	α	τ^{-1}	τ
1950-55	3.81	2.222	2.75	2.38	1.15	6.54	0.07	0.05	19.56
1955-60	4.02	2.150	2.72	2.38	1.14	6.47	0.07	0.05	19.65
1960-65	4.00	2.063	2.65	2.36	1.12	6.24	0.06	0.05	20.02
1965-70	3.57	2.186	2.44	2.17	1.13	5.31	0.06	0.05	21.69
1970-75	3.30	1.936	2.44	1.85	1.32	4.51	0.14	0.04	23.55
1975-80	2.74	1.334	2.11	1.51	1.40	3.18	0.17	0.04	28.02
1980-85	2.58	1.606	2.04	1.23	1.66	2.50	0.25	0.03	31.59
1985-90	2.49	1.602	1.98	1.00	1.98	1.97	0.34	0.03	35.62
1990-95	2.49	1.484	1.95	0.85	2.29	1.66	0.41	0.03	38.76
1995-2000	2.45	1.387	1.87	0.76	2.44	1.43	0.45	0.02	41.89
2000-05	2.41	1.487	1.78	0.73	2.44	1.29	0.45	0.02	43.95
2005-10	2.49	1.747	1.80	0.72	2.50	1.29	0.46	0.02	44.00
2010-15	2.45	1.455	1.73	0.75	2.33	1.29	0.42	0.02	43.99

Table 2g: World

Years	*FR*	*GR*	*BR*	*DR*	*x*	*y*	α	τ^{-1}	τ
1950-55	4.95	1.816	3.686	1.871	1.97	6.90	0.07	0.05	19.04
1955-60	4.89	1.829	3.541	1.714	2.07	6.07	0.07	0.05	20.30
1960-65	4.91	1.851	3.462	1.612	2.15	5.58	0.06	0.05	21.16
1965-70	4.85	2.069	3.363	1.296	2.59	4.36	0.06	0.04	23.95
1970-75	4.45	1.958	3.132	1.175	2.67	3.68	0.14	0.04	26.06
1975-80	3.84	1.767	2.830	1.064	2.66	3.01	0.17	0.03	28.82
1980-85	3.59	1.763	2.771	1.010	2.74	2.80	0.25	0.03	29.89
1985-90	3.39	1.744	2.698	0.955	2.82	2.58	0.34	0.03	31.14
1990-95	3.04	1.523	2.447	0.925	2.65	2.26	0.41	0.03	33.25
1995-2000	2.79	1.339	2.236	0.897	2.49	2.01	0.45	0.03	35.30
2000-05	2.62	1.216	2.085	0.869	2.40	1.81	0.45	0.03	37.15
2005-10	2.52	1.162	2.001	0.839	2.38	1.68	0.46	0.03	38.59
2010-15	2.45	1.096	1.915	0.819	2.34	1.57	0.42	0.03	39.92

(*) Source UN World Population Prospects: The 2010 Revision (updated April 2011). For 2015 we use the Medium-fertility variant 2010-2100.

Table 3: Together with Table 6.

Table 4: (*DR*) vs. $\left(LE_f\right)^{-1}$

	World		Africa		Europe		Asia		N. Amer.		L. Amer.	
Year	*DR*	*Lf-1*	*DR*	*Lf-1*	*DR*	*Lf-1*	*DR*	*Lf-1*	*DR*	*Lf-1*	*DR*	*Lf-1*
1950-55	1.871	0.021	2.710	0.025	1.073	0.015	2.570	0.023	1.130	0.014	2.380	0.019
1955-60	1.714	0.020	3.210	0.024	1.073	0.014	2.350	0.022	1.320	0.014	2.378	0.018
1960-65	1.612	0.019	3.260	0.023	1.041	0.014	2.120	0.021	1.130	0.014	2.357	0.017
1965-70	1.296	0.017	3.140	0.022	1.150	0.014	1.950	0.018	1.010	0.013	2.173	0.016
1970-75	1.175	0.017	2.880	0.021	1.080	0.013	1.640	0.017	1.140	0.013	1.849	0.016
1975-80	1.064	0.016	2.570	0.020	1.030	0.013	1.330	0.016	0.470	0.013	1.509	0.015
1980-85	1.010	0.016	2.390	0.019	1.060	0.013	0.960	0.016	0.470	0.013	1.230	0.015
1985-90	0.955	0.015	2.180	0.019	0.970	0.013	0.760	0.015	0.530	0.013	0.997	0.014
1990-95	0.925	0.015	1.810	0.019	1.065	0.013	0.690	0.015	0.860	0.013	0.852	0.014
1995-00	0.897	0.015	1.910	0.019	1.080	0.013	0.679	0.015	0.850	0.013	0.764	0.014
2000-05	0.869	0.015	1.760	0.018	1.140	0.013	0.694	0.014	0.880	0.013	0.728	0.013
2005-10	0.839	0.014	1.150	0.018	1.180	0.013	0.683	0.014	0.840	0.012	0.718	0.013

Table 5:

	World		*Europe*	
Years	(*BR*)	(*DR*)	(*BR*)	(*DR*)
1950-55	3.69	1.87	2.14	1.07
1955-60	3.54	1.71	2.07	1.07
1960-65	3.46	1.61	1.91	1.04
1965-70	3.36	1.30	1.67	1.15
1970-75	3.13	1.18	1.56	1.08

1975-80	2.83	1.06	1.48	1.03
1980-85	2.77	1.01	1.43	1.06
1985-90	2.70	0.96	1.37	0.97
1990-95	2.45	0.92	1.15	1.07
1995-00	2.24	0.90	1.02	1.08
2000-05	2.08	0.87	1.02	1.14
2005-10	2.00	0.84	1.08	1.18
2010-15	1.73	0.82	0.92	1.13
2015-20	1.53	0.82	0.81	1.14
2020-25	1.39	0.82	0.74	1.16
2025-30	1.33	0.84	0.74	1.18
2030-35	1.27	0.87	0.75	1.22
2035-40	1.19	0.91	0.76	1.27
2040-45	1.12	0.95	0.745	1.31

Source: UN.

Table 3: World population 1950-2050.

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015	2020	2025	2030	2035	2040
$P(t)_{cal}$	2.59	2.78	2.35	2.60	3.29	3.73	4.23	4.83	5.27	5.72	6.20	6.58	6.80	7.08	7.27	7.42	7.53	7.61	7.68
$P(t)_{UN}$	2.53	2.77	3.02	3.33	3.69	4.06	4.34	4.83	5.26	5.67	6.07	6.40	6.69	7.22	7.48	7.69	7.87	8.01	8.10

Calculated population (P_{cal} using Eq. (1)) and UN data (P_{UN}). Fitting parameters $\left(t \geq 1985\right)$: $P_{RL}\left(1960\right) \simeq 2.0 \times 10^{9}$; $t_{\mathrm{inf}} = 1985$; $\tau^{*} \simeq \tau / \cosh\alpha = 30$ years; $\frac{1}{2}\Delta P_{\mathrm{m}} \tanh\alpha = 2.93 \times 10^{9}$.

Table 6: A comparison of Malthusian, (*IRE*) and UN projections 1950-2040.

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015	2020	2025	2030	2035	2040
$P(t)_{UN}$	2.53	2.77	3.04	3.33	3.70	4.08	4.45	4.86	5.31	5.73	6.12	6.51	6.90	7.22	7.48	7.69	7.87	8.01	8.10
$P(t)_{cal}$ [1]	2.59	2.78	3.02	3.31	3.65	4.04	4.48	4.93	5.38	5.82	6.21	6.55	6.84	7.08	7.27	7.42	7.53	7.61	7.68
$P(t)_{Malthusian}$	2.53	2.77	3.04	3.33	3.70	4.08	4.45	4.86	5.32	5.81	6.36	6.95	7.60	8.30	9.08	9.93	10.85	11.87	12.97

Figures

Figure 1: (*BR*) vs. (*FR*) by continents and the world.

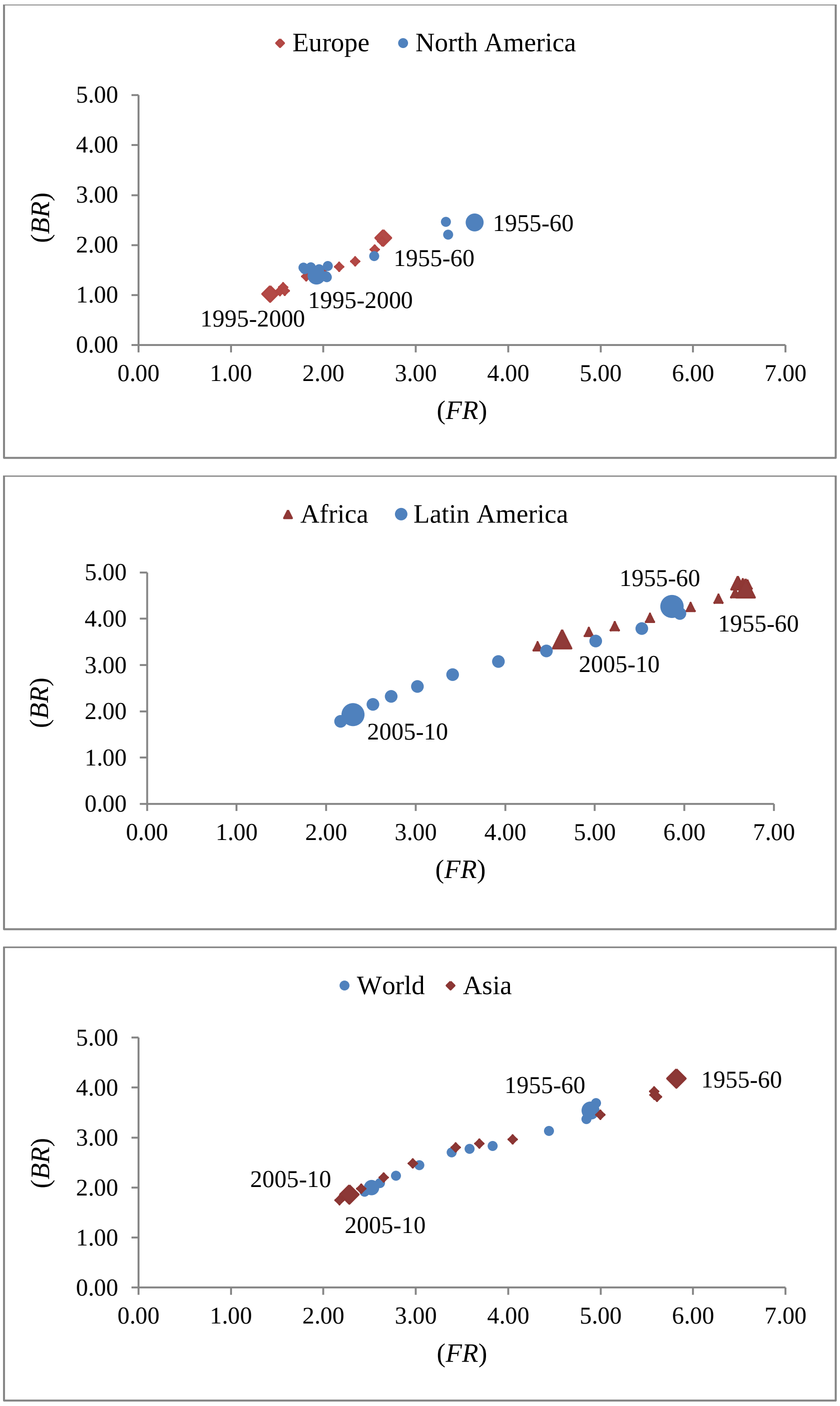


Figure 2a: $x(t)$

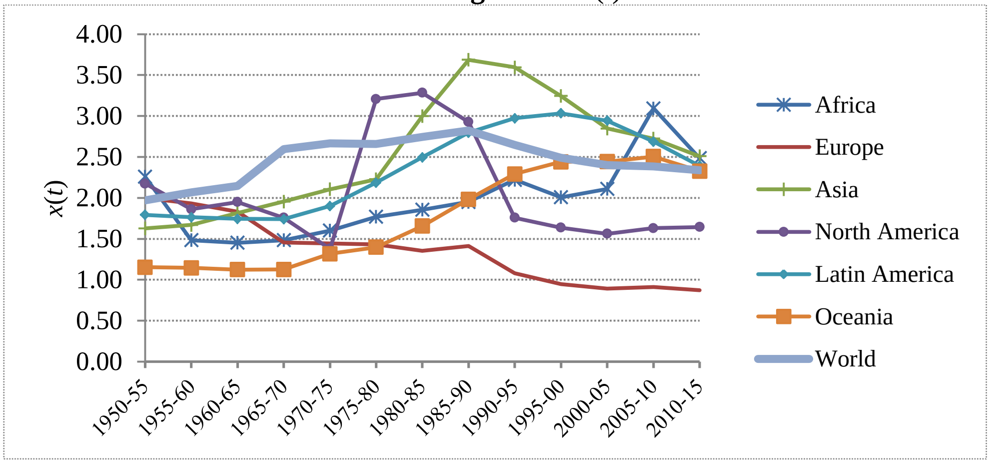


Figure 2b: $\tau(t)$

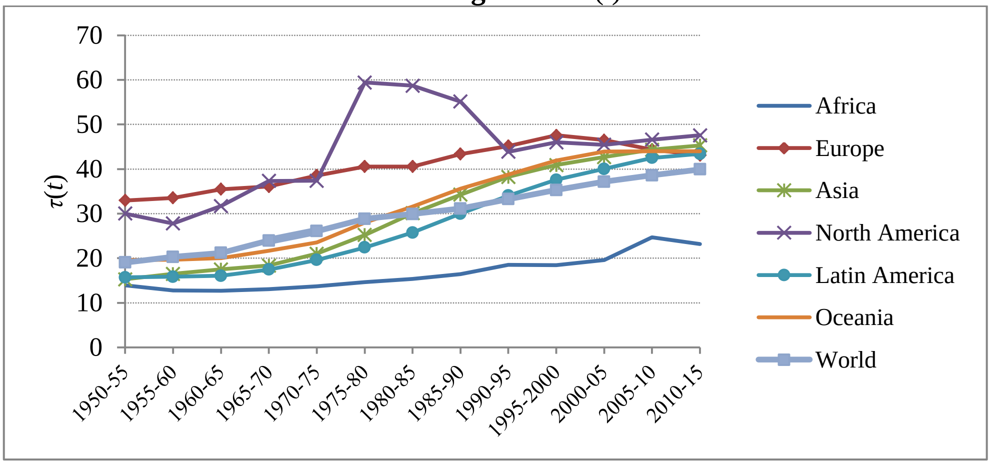
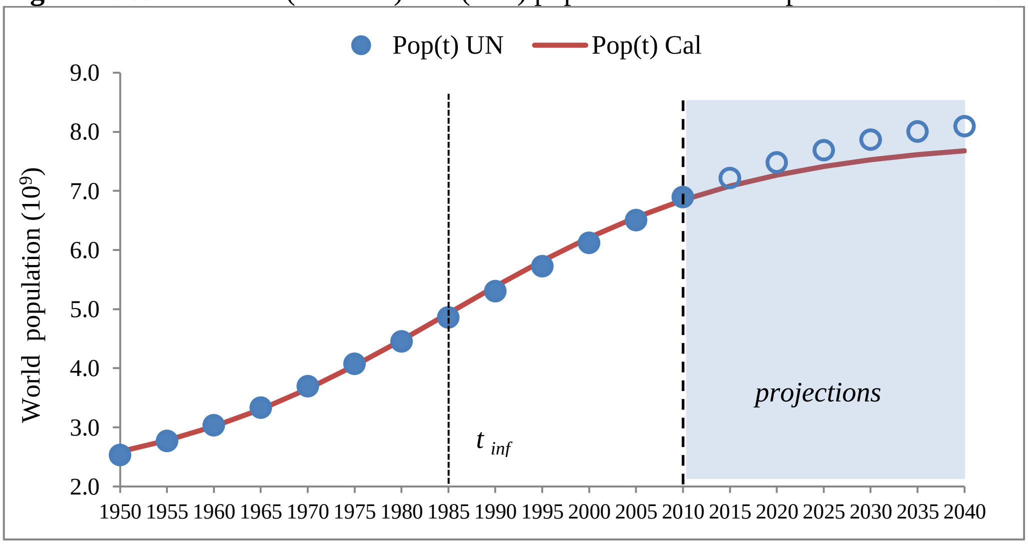


Figure 3: World actual (UN data) and (IRE) population fit for the period 1960-2010.



Source: UN data and (IRE) model projection -Eq(1). $\frac{1}{2}\Delta P_m\left(\tanh\alpha\right) = 2.93$, $\tau = 32$, $t_{inf} = 1985$, $P_{RL} = 2.0$. Note: The shadow area corresponds to projections.

Figure 4: (DR) vs. $\left(LE_f\right)^{-1}$

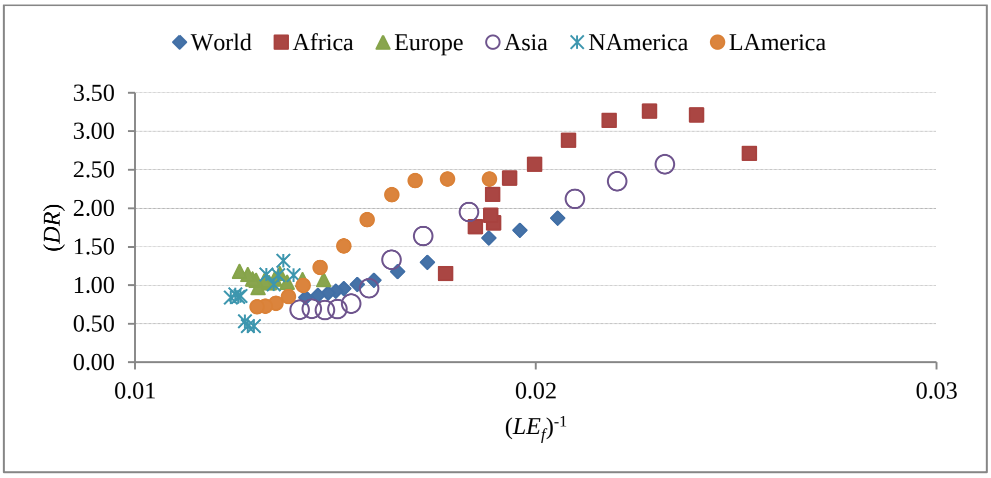


Figure 5: (BR) & (DR) for the World and Europe 1950-2045.

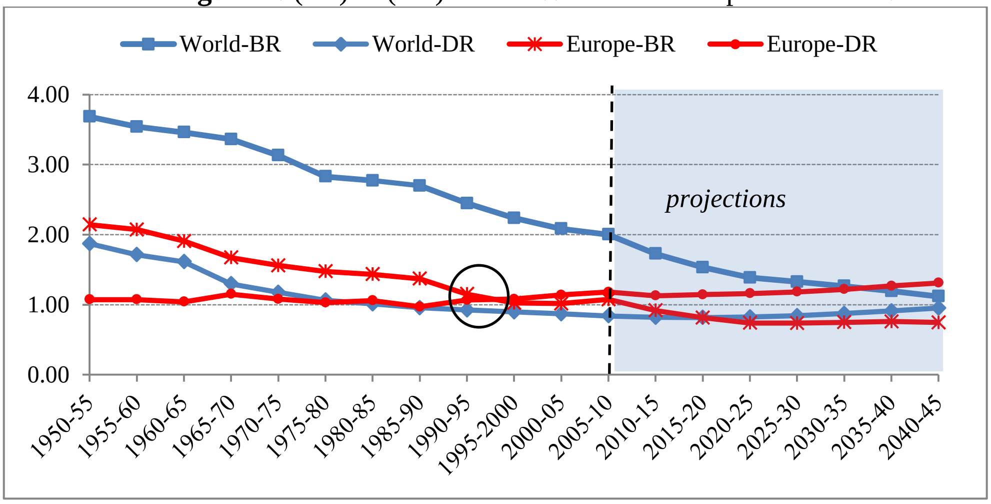
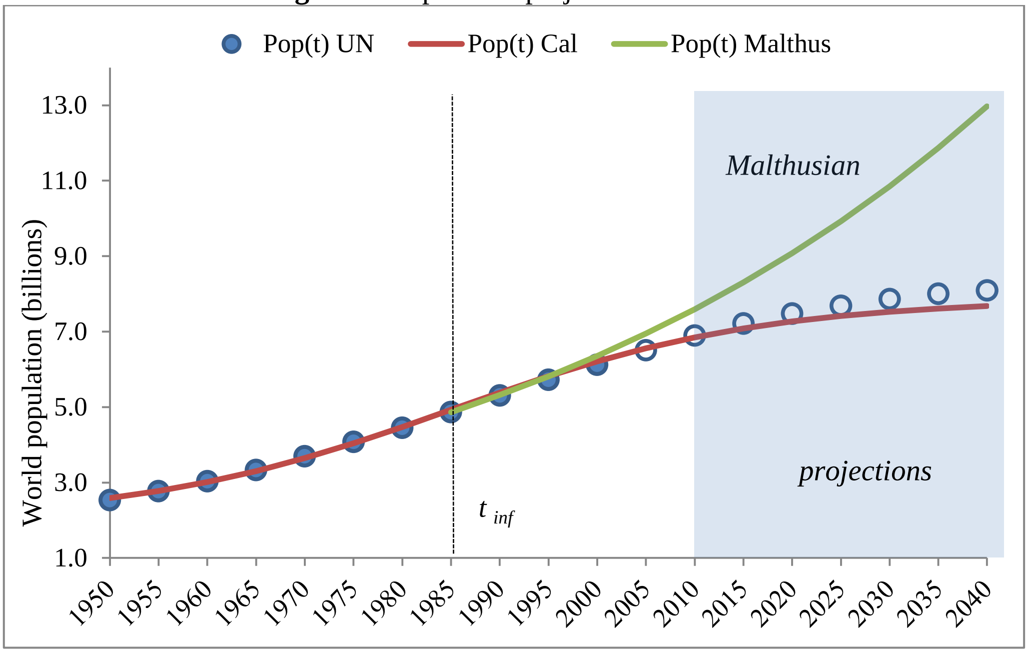


Figure 6: Population projections 1950-2040.



Source: UN data and (IRE) model projection –Eq. (1). $\frac{1}{2}\Delta P_m(\tanh\alpha) = 2.93$, $\tau = 32$, $t_{inf} = 1985$, $P_{RL} = 2.0$. Note: The shadow area corresponds to projections.

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